Solutions

3.3: Applications of Systems of Linear Equations

Example 1. The Arctic Juice Company makes three juice blends: PineOrange, using 2 quarts of pineapple juice and 2 quarts of orange juice per gallon; PineKiwi, using 3 quarts of pineapple juice and 1 quart of kiwi juice per gallon; and OrangeKiwi, using 3 quarts of orange juice and one quart of kiwi juice per gallon. Each day the company has 800 quarts of pineapple juice, 650 quarts of orange juice and 350 quarts of kiwi juice available. How many gallons of each blend should it make each day if it wants to use up all of the supplies?

	Pine Orange (x)	Pine Kiwi (y)	OrangeKiwi (z)	Total
Pineapple juice (qt)	2	3	0	800
Orange juice (qt)	2	0	3	650
Kiwi Juice (qt)	0	1	1	350

Augmented Matrix

RREF

$$\left(\begin{array}{ccc|c} 2 & 3 & 0 & 800 \\ 2 & 0 & 3 & 650 \\ 0 & 1 & 1 & 350 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 200 \\ 0 & 0 & 1 & 150 \end{array}\right)$$

So $(x,y,z) = (100, 200, 150)$

The Arctic Juice Company should make

100 gallons of ~~the~~ PineOrange,

200 gallons of ~~the~~ PineKiwi,

and 150 gallons of OrangeKiwi.

Example 2. A new airline has recently purchased a fleet of Airbus A330-300s, Boeing 767-200ERs, and Boeing Dreamliner 787-9s to meet an estimated demand for 4,800 seats. The A330-300s seat 320 passengers and cost $200 million each, the 767-200ERs each seat 250 passengers and cost $125 million each, while the Dreamliner 787-9s seat 275 passengers and cost $200 million each. The total cost of the fleet, which had twice as many Dreamliners as 767s, was $3,100 million. How many of each type of aircraft did the company purchase?

	A330-300	767-200ER	787-9 Dreamliner	Total
Capacity	320	250	275	4,800
Cost (millions)	200	125	200	3,100

We also know $z = 2y$ or $2y - z = 0$.

Augmented Matrix

$$\left(\begin{array}{ccc|c} 320 & 250 & 275 & 4{,}800 \\ 200 & 125 & 200 & 3{,}100 \\ 0 & 2 & -1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 8 \end{array}\right)$$

RREF

So $(x, y, z) = (5, 4, 8)$

The airline ordered 5 A330-300s,
4 767-200ERs,
and 8 Dreamliner 787-9s.

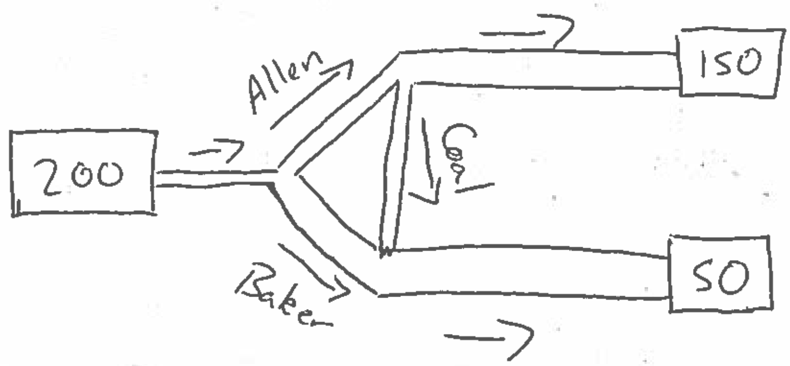
Example 3. Traffic through downtown Urbanville flows through the one-way system shown below. Traffic counting devices installed in the road (shown as boxes) count 200 cars entering town from the west each hour, 150 leaving town on the north each hour, and 50 leaving town on the south each hour.

a. From this information, is it possible to determine how many cars drive along Allen, Baker, and Coal streets every hour?

b. What is the maximum possible traffic flow along Baker Street?

c. What is the minimum possible traffic flow along Allen Street?

d. What is the maximum possible traffic flow along Coal Street?



Unknowns: x = # of cars per hour on Allen Street
y = # of cars per hour on Baker Street
z = # of cars per hour on Coal Street.

Equations: $200 = x+y$
$150 = x-z$
$50 = y+z$

Augmented Matrix

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 200 \\ 1 & 0 & -1 & 150 \\ 0 & 1 & 1 & 50 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -1 & 150 \\ 0 & 1 & 1 & 50 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

RREF

So z is arbitrary, $x-z=150$ and $y+z=50$.

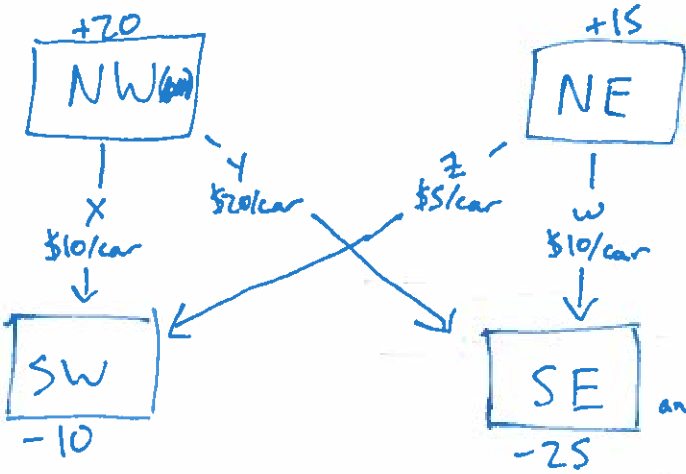
(a) No, we cannot by RREF.

(b) Maximum for y is 50.

(c) Minimum for x is 150.

(d) Maximum for z is 50.

Example 4. A car rental company has four locations in the city: Southwest, Northeast, Southeast, and Northwest. The Northwest location has 20 more cars than it needs, and the Northeast location has 15 more cars than it needs. The Southwest location needs 10 more cars than it has, and the Southeast locations needs 25 more cars than it has. It costs \$10 (in salary and gas) to have an employee drive a car from Northwest to Southwest. It costs \$20 to drive a car from Northwest to Southeast. It cost \$5 to drive a car from Northeast to Southwest, and it costs \$10 to drive a car from Northeast to Southeast. If the company will spend a total of \$475 rearranging its cars, how many cars will it drive from each of Northwest and Northeast locations to each of Southwest and Southeast locations?



We know

$x+y=20$

$z+w=15$

$x+z=10$

$y+w=25$

and $10x+20y+5z+10w=475$

Augmented Matrix

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 1 & 15 \\ 1 & 0 & 1 & 0 & 10 \\ 0 & 1 & 0 & 1 & 25 \\ 10 & 20 & 5 & 10 & 475 \end{array}\right) \longrightarrow$$

RREF

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

So $(x,y,z,w)=(5,15,5,10)$

5 cars from NW to SW

15 cars from NW to SE

5 cars from NE to SW

10 cars from NE to SE.

Bonus Question: Is \$475 the minimum amount of money needed to be spent rearranging cars in Example 4?

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 1 & 15 \\ 1 & 0 & 1 & 0 & 10 \\ 0 & 1 & 0 & 1 & 25 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 1 & 25 \\ 0 & 0 & 1 & 1 & 15 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x=w-5$

$y=25-w$

$z=15-w$

w is arbitrary

Cost $=10x+20y+5z+10w$

$\Rightarrow\ =10(w-5)+20(25-w)+5(15-w)+10w$

$=525-5w$

w is between 5 and 15.

when $w=15$, $(x,y,z,w)=(10,10,0,15)$ w/ cost \$45